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ABSTRACT

A method is presented for analyzing connections with curved inner flanges commonly used in rigid frame construction. The method is based on simple plastic theory. The bending capacity and the stability of the connection are considered. Also, the effect of cross-bending of the flanges, shear, and axial force is discussed.

Design procedures for proportioning a connection with a curved inner flange are presented. The design of the stiffeners is included. Examples illustrating the design method are presented.

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1. INTRODUCTION

It is the purpose of this study to present a plastic method of analysis for connections having curved inner flanges. A detailed description of haunched connections with straight-tapered knees and a review of related literature may be found elsewhere[1].

The sequence of analyzing various aspects of a curved knee is as follows. A solution is first obtained based on simple plastic theory, in which the most critical section within a curved knee is determined. The influence of shear and axial force on the plastic solution is then discussed. Also, the stability of the compression flange is treated; the relationships between the maximum unsupported length, the thickness of the compression flange, and the geometry of the knee are analyzed. Finally a recommended design procedure is presented, and design examples are given.

The theory developed in this study has been verified experimentally[2]. The observed results showed good agreement with those predicted by the theories presented herein.

2. PLASTIC ANALYSIS

This section presents a plastic method of analysis for connections having curved inner flanges. A typical connection is shown in Fig. 1. A solution based on simple plastic theory is developed first. The location of the most critical section within the haunch is obtained by maximizing the solution. The influence of axial force and shear on the plastic solution is also treated.

2.1 Solution by Simple Plastic Theory

The assumptions and conditions upon which this analysis is based are as follows:

1. Plane sections remain plane after bending; thus the bending strains are proportional to the distance from the neutral axis.
2. The idealized stress-strain diagram is assumed, and the behavior of fibers in bending is the same as in compression and tension.
3. Equilibrium between the applied loads and moments and the resulting stress distribution exists in order that

Normal force: $$P = \int_A \sigma_y \, dA \qquad (1)$$

Moment: $$M = \int_A \sigma_y \, y \, dA \qquad (2)$$

4. The force in the curved flanges is approximately uniform along its length. (Since the depth of the haunch increases rapidly after β is greater than $\frac{\pi}{8}$

as shown in Fig. 1, considerable variation is possible in the curved flange force. However, since a plastic hinge forms within the curved portion of the knee, the assumption appears reasonable when the plastic moment value is reached)

A typical knee with a curved inner flange is shown in Fig. 1, and the stress distribution is indicated in Fig. 2.

The distance c for location of the neutral axis is determined by considering the force equilibrium in the horizontal direction:

$$\sigma_y \, bt + \sigma_y \, (c-t)w = \sigma_y \, bt \cos\beta + \sigma_y \, (d_z-c-t)w$$

Thus

$$c = \frac{d_z}{2} - \frac{bt}{2w} (1-\cos\beta) \qquad (3)$$

However, it is noted that if the thickness of the lower flange is increased by the factor $\frac{1}{\cos\beta}$, a condition of symmetry is obtained which yields $c = \frac{d_z}{2}$. This substantially reduces the unwieldly expressions which would otherwise occur. If the location of the most critical section is near the rolled section, for small angles of β there is very little, if any, increase necessary. Hence, it would be possible to design the curved haunched connection as though it were a beam with parallel flanges.

When the position of the neutral axis is known, the value of the full plastic moment may be determined by Eq.(2).

Therefore:

$$M_p = \int_A \sigma_y \, ydA_z = \sigma_y \, Z_z \tag{4}$$

where $Z_z = \int_A ydA_z$ = plastic modulus. If the compression flange has been increased by the factor $\frac{1}{\cos\beta}$, then

$$Z_z = bt(d_z - t) + \frac{w}{4}(d_z - 2t)^2 \tag{5}$$

Since the web thickness w and the flange width b are usually maintained equal to those of the adjacent rolled sections, and the depth d_z of the critical point is fixed by geometry, it is possible to obtain the required thickness of haunch flange.

2.2 Location of the Critical Section in a Curved Knee

If the position of the critical section within the curved knee is known, the theoretical analysis would be greatly simplified. It is the purpose of this section to locate this position.

As in the case of the tapered knee[1], a linear distribution of moment will be assumed between the inflection point and any point within the haunch. Fig. 1 indicates the general proportions of the connection. The flange thickness is determined by equating the moment at some unknown point within the haunch to the plastic moment,

$$\frac{M_{pr}}{a}(a + R\sin\beta) = \sigma_y \, Z_z \tag{6}$$

where M_{pr} = applied moment at the junction of the connection and the rolled section. However, M_{pr} equals $\sigma_y \, Z_r$ where Z_r is the plastic modulus of the adjacent rolled sections.

Hence:

$$\frac{\sigma_y}{a} Z_r(a+R\ \sin\beta) = \sigma_y\left(bt\left[(d-t)+R(1-\cos\beta)\right] + \frac{w}{4}\left[(d-2t)+R(1-\cos\beta)\right]^2\right)$$

or

$$t = \frac{d+R(1-\cos\beta) - \sqrt{d+R(1-\cos\beta)\ ^2\ \left(\frac{b}{b-w}\right) - \frac{4Z_r(a+R\ \sin\beta)}{a(b-w)}}}{2} \qquad (7)$$

It is possible to maximize the thickness of the haunch flange with respect to the angle β.

Thus:

$$\frac{\partial t}{\partial\beta} = \frac{1}{2} R\ \sin\beta - \frac{1}{4}\ \frac{2\left[d+R(1-\cos\beta)\right]\ \left(\frac{b}{b-w}\right) R\sin\beta - \frac{4Z_r}{a(b-w)} R\ \cos\beta}{\sqrt{\left[d+R(1-\cos\beta\ )\right]^2\ \left(\frac{b}{b-w}\right) - \frac{4Z_r(a+R\ \sin\beta\ )}{a(b-w)}}} \qquad (8)$$

If values of a and R are assumed for typical connections, it is possible to plot the rate of change of thickness with respect to the angle β for various sizes of rolled sections. The values of d, b, w, and Z_r in Eq.(8) are those of the rolled section. A plot of Eq.(8) is shown in Fig. 3. Since the thickness t is a maximum when zero slope is approached from a positive sense, the critical angle is approximately 12 deg. Values of a and R used in the computations for the curves in Fig. 3 are indicated. Except for unusual cases where shear is very large, it is considered that the critical section can always be taken at β = 12 deg.

If the angle of intersection between the girder and column is increased, the assumption of a linear moment diagram between the points of tangency and the critical section will not be affected. Therefore, the critical section for rectangular and gable frame knees will always be approximately 12 deg. from the points of tangency.

Test results of various curved knees having a variety of proportions have all seemed to indicate that the critical section is within the immediate vicinity of a point 12 deg. from the point of tangency. It is therefore concluded that this is a reasonable solution. Also, since the critical angle β is so small, the compression flange only needs to be increased by the factor 1.02. For all practical purposes this can be ignored, and hence the required thickness of the haunch flanges can be readily obtained.

2.3 Influence of Shear and Axial Thrust

Since the area of a section within the haunch is always greater than a comparable section in the adjacent rolled section, it may be concluded that the effect of the axial force and the shear in the knee will be less critical than in the rolled section. Also, previous tests have shown that shear has little influence on the maximum bending strength of most structural members(3). Thus, as long as the critical section is equal to or greater than the rolled section, the effect of axial force and shear may be neglected.

3. STABILITY OF THE COMPRESSION FLANGE

If a connection is to develop its fully plastic capacity, premature failure due to stability considerations must be prevented. Both local and lateral stability are treated in this section.

3.1 Lateral Buckling of the Compression Flange

Due to the curved shape of the compression flange the strain energy approach was used. The strain energy of lateral bending was equated to the work done by the flange force when deformation occurs. Before formulating the energy of the system, it is necessary to state the conditions and assumptions made.

1. The curved compression flange buckles independently of the remaining component parts of the cross section. The resistance to buckling offered by the adjacent web is assumed negligible.

2. The curved flange is assumed to have a uniform stress distribution corresponding to the yield stress over its area and is assumed to have reached the strain-hardening state along its length.

3. The compression flange is simply supported at its connection with the rolled section.

4. The flange must buckle normal to the plane of the haunch. (Twisting is assumed not to occur in the compression flange because of the restraint of the web. A solution which includes the effect of twisting is contained in Timoshenko's Theory of Elastic Stability, p. 285)

5. The forces restraining the flange from buckling in the plane of bending are transmitted by the web. It is assumed in the derivation that the directions of the loads do not change during buckling, and that they are displaced laterally only, thus remaining parallel to their initial direction. Hence, no work is performed by the forces restraining the flange during this translation.

The general case of the curved flange is shown in Fig. 4. From the assumption of simply supported ends the deflection curve of the slightly buckled flange may be expressed by the equation

$$y = \sin \frac{\pi s}{R\alpha} \qquad (9)$$

where s is taken as the arc length

The displacement of the load P during buckling produces the corresponding work.

$$T = \frac{P}{2} \int_{-\frac{S}{2}}^{\frac{S}{2}} \left(\frac{dy}{ds}\right)^2 ds \qquad (10)$$

Hence

$$T = \frac{P}{2} \left(\frac{\pi}{R\alpha}\right)^2 \int_{-\frac{R\alpha}{2}}^{\frac{R\alpha}{2}} \cos^2 \frac{\pi s}{R\alpha}\, ds$$

$$= \frac{P}{2} \left(\frac{\pi}{R\alpha}\right)^2 \cdot \frac{R\alpha}{2} \qquad (11)$$

Likewise, the corresponding energy of bending U is given by

$$U = \frac{E_{st} I_x}{2} \int_0^S \left(\frac{d^2 y}{ds^2}\right)^2 ds \qquad (12)$$

Hence

$$U = \frac{E_{st}I_x}{2} \left(\frac{\pi}{R\alpha}\right)^4 \int_0^{R\alpha} \sin^2 \frac{\pi s}{R\alpha} \, ds$$

$$= \frac{E_{st}I_x}{2} \left(\frac{\pi}{R\alpha}\right)^4 \cdot \frac{R\alpha}{2} \qquad (13)$$

The critical value of the load at which equilibrium changes from stable to unstable with respect to lateral buckling in the strain-hardening range is determined from the equation

$$U = T \qquad (14)$$

Hence

$$\frac{E_{st}I_x}{2} \left(\frac{\pi}{R\alpha}\right)^4 \frac{R\alpha}{2} = \frac{P}{2} \left(\frac{\pi}{R\alpha}\right)^2 \frac{R\alpha}{2} \qquad (15)$$

Since $I_x = r_x^2 A_f$ and $P = \sigma_y A_f$, the critical length of the curved flange can then be expressed as

$$\left(\frac{R\alpha}{r_x}\right)_{cr} = \pi \sqrt{\frac{E_{st}}{\sigma_y}} \qquad (16)$$

The strain-hardening modulus E_{st} will be taken as 900 ksi[4]. Hence, Eq.(16) can now be expressed as

$$\left(\frac{R\alpha}{r_x}\right)_{cr} = \pi \sqrt{\frac{900}{33}} = 16.5 \qquad (17)$$

Since

$$r_x = \sqrt{\frac{I_x}{A_f}} = \sqrt{\frac{1}{tb} \cdot \frac{tb^3}{12}} = \frac{b}{\sqrt{12}}$$

Then

$$(R\alpha)_{cr} = 16.5\frac{b}{\sqrt{12}} = 4.8 \text{ b} \qquad (18)$$

For square-corner curved knees used in portal frames a maximum allowable radius of the curved flange may be formulated. If lateral support is provided at points A, B, and C in Fig. 5, it can be assumed that the compression flange is forced to buckle in the second mode. Hence, the critical length can then be expressed as

$$R \frac{\pi}{4} = 4.8 \text{ b}$$

Therefore, the maximum allowable radius of the curved inner flange is

$$R = 6b \qquad (19)$$

A larger radius may be permitted if the number of points of lateral support is increased or the magnitude of r_x is increased in Eq.(17). The radius of gyration can be increased by widening the compression flange or incorporating a special shape which will greatly increase r_x. It is also possible to increase the flange thickness to achieve stability when a radius larger than 6b is desirable. It has been shown[1] that the increase in flange thickness Δt required for any radius is

$$\Delta t = 0.11 \left(\frac{R\alpha}{b} - 4.8\right) \qquad (20)$$

Therefore

$$t_t = t_c = (1 + \Delta t) \ t \qquad (21)$$

where t is the flange thickness determined from Eq.(7) using the critical dimensions of $\beta = 12°$ and $R = 6b$.

For knees used in gabled frames it will be necessary to determine the angle $\frac{\alpha}{2}$ between the points of lateral support before the critical buckling length and hence the maximum allowable radius can be obtained. However, it can be seen from Fig. 6 that the angle $\frac{\alpha}{2}$ will always be less than $\frac{\pi}{4}$ in gabled frames. Thus the maximum allowable radius will be greater than that allowed for knees whose members intersect at right angles.

3.2 Effect of Warping

One of the important assumptions made in the previous section was that the fiber stress along the curved flanges was approximately at the yield value throughout its length. This stress produces a radial component which tends to bend the curved flange across the web plate. This problem was recognized in the elastic analysis of curved flanges, and a solution was developed by Bleich[5]. It was realized in elastic analysis that the transverse displacement of the flanges could greatly influence the distribution of longitudinal stress over the cross section. Therefore, when the plastic analysis of a curved knee was considered, it was felt necessary to investigate the effect of this phenomenon on the full plastic moment and geometry of the section.

In order to complete the analysis, it is first necessary to calculate the magnitude of the transverse forces acting in the curved flange of the knee. From the first assumption in Section 3.1 the force in a fiber of unit width with radius of curvature R becomes $\sigma_z t$. From Fig. 7 the transverse force in the length ds becomes

$$p = \sigma_z t \, d\alpha \tag{22}$$

The transverse force per unit of length is

$$\frac{p}{ds} = \sigma_z t \frac{d\alpha}{ds} = \frac{\sigma_z t}{R} \qquad (23)$$

A strip of flange having a unit width and a length of $\frac{b}{2}$ can then be subjected to this uniform transverse force as shown in Fig. 8. This can be examined as an approximation of the true behavior of the compression flange of a curved knee when subjected to forces tending to close the knee. It is assumed that the symmetry of the section causes a strip of unit width of the flange to act as a cantilever rigidly fixed at the base and subjected to a uniform load of $\frac{\sigma_z t}{R}$.

From Fig. 8 the maximum moment is

$$M = \sigma_y \frac{t}{R} \frac{b^2}{8} \qquad (24)$$

The plastic modulus of a one-inch strip of curved flange assumes the value

$$Z = \frac{t^2}{4} \qquad (25)$$

The limiting value of the warping stress, which is equal to the yield stress, can now be equated

$$\sigma_y = \frac{M}{Z} = \frac{\sigma_y t b^2}{8R} \cdot \frac{4}{t^2}$$

or

$$\sigma_y = \frac{\sigma_y b^2}{2Rt} \qquad (26)$$

Hence:

$$\frac{b^2}{Rt} \lessapprox 2 \qquad (27)$$

It is interesting to note that Eq.(27) corresponds identically with Rule 7 presented by Griffiths[6] in his "Design Rules" for the proportioning of rigid frame knees. One other point to note is that this critical condition is maximum only at the junction of flange and web. The transverse stress varies from zero on the flange tips to the assumed maximum value σ_y.

3.3 Radial Stiffeners in Curved Knees

It was recognized in elastic design that it was desirable to provide stiffeners at the mid point and at or near the extremities of a curved knee[6]. It is evident that they should be used also in plastic design to prevent undue shear deformation and premature web buckling. Unfortunately, no "precise" mathematical or empirical solution exists for the design of such members. It is not proposed in this section to present a rigorous mathematical solution to this problem. The analysis and rules presented are an approximate means of determining the required stiffener area.

The following assumptions are made:

1. The diagonal stiffener at the corner must resist a force due to the curved flange.

2. The force transmitted is the component of a force equal to the plastic flange force which passes through the midpoint of the curved flange and the points of tangency at the rolled section (Figs. 9 and 10).

The above assumptions are arbitrary, but it is believed that they result in a conservative solution. The actual forces transmitted by the curved flange into the diagonal stiffener are no doubt much smaller since much of the radial force is taken by the web of the haunch. However, it is desirable to have some way of proportioning the diagonal stiffener, which is primarily used to prevent the haunch web from buckling.

For a rectangular portal frame having a curved inner flange as shown in Fig. 9, the required stiffener thickness is determined as follows. The force F_s acting on the diagonal stiffener is, from assumption 2, equal to

$$F_s = 2\,\sigma_y\,A_f\,\sin 22.5° \qquad (28)$$

The required stiffener area is then obtained as

$$A_s = \frac{F_s}{\sigma_y}$$

$$A_s = 2\,A_f\,\sin 22.5° \qquad (29)$$

If the width of the stiffener is maintained the same as the width of the haunch flange, the required stiffener thickness becomes

$$t_s = 2t\,\sin 22.5° = 0.766\,t \qquad (30)$$

Generally, for ease in fabrication and as a matter of practicality, the stiffener could be cut from material with the same thickness as that of the flanges.

For connections proportioned for use in gabled frames in which the girder intersects the column at an angle larger than a right angle, the following solution for the required stiffener thickness is obtained. The component of the flange forces resisted by the diagonal stiffener can be obtained from Fig. 10.

Hence,

$$F_s = 2\,\sigma_y A_f \sin\left(22.5° - \frac{1}{4}\theta\right) \qquad (31)$$

Thus, the required stiffener area is

$$A_s = 2A_f \sin\left(22.5° - \frac{1}{4}\theta\right) \qquad (32)$$

Again, this can be simplified further by making the stiffener width the same as the haunch flanges. The required stiffener thickness that is obtained is

$$t_s = 2t \sin\left(22.5° - \frac{1}{4}\theta\right) \qquad (33)$$

For most gabled frames of practical proportions such that the roof pitch is not excessive, the influence of the gable angle can be neglected.

4. DESIGN RECOMMENDATIONS AND ILLUSTRATIVE EXAMPLES

The recommendations presented here were evolved from the preceding sections. The examples are included to illustrate the application of the design procedure to a hypothetical case.

4.1 Recommended Design Procedure

A curved knee may be proportioned on the following basis:

1. The critical design sections are taken at the points of tangency and at sections 12 deg. from the points of tangency.

2. The size of rolled section required at the points of tangency would be selected by simple plastic theory. The flange width and web thickness of the haunch is usually maintained the same as in the rolled section.

3. The required flange thickness of the inner and outer flanges at the critical section 12 deg. within the haunch can be determined by Eq.(7)

$$t = \frac{d_z - \sqrt{d_z^2 \left(\frac{b}{b-w}\right) - \frac{4\,M_z}{\sigma_y (b-w)}}}{2}$$

where the dimensions and moment M_z are taken at the critical section.

4. Because of local buckling the value of t must be such that $\frac{b}{t}$ does not exceed 17[7].

5. The maximum allowable radius of the curved flange for a rectangular portal frame knee is R = 6b when positive

lateral support is provided for the compression (curved) flange at the midpoint and at or near the points of tangency. If it is desirable to increase the radius of the curved flange, more lateral support must be provided such that the arc length between support points does not exceed 5b. Another method of increasing the radius without increasing the points of lateral support is to increase the radius of gyration of the curved flange so that

$$\left(\frac{R\alpha}{r_x}\right)_{cr} = 16.5$$

If it is undesirable to provide intermediate points of support, the thickness of the flanges within the haunch can be increased to control the strains by utilizing Eqs.(20) and (21).

6. For knees in gabled frames, where α is the angle between the points of lateral support, the maximum allowable radius can be found from Eq. (18).

7. The relationship between the width of the curved flange to its thickness must such that

$$\frac{b^2}{2Rt} \leqq 1$$

If this is exceeded, it is necessary to increase the thickness of the flange or to provide short stiffeners along the curved flange.

8. Stiffeners should be provided at the midpoint of the curved flange and at or near the points of tangency. The stiffener at the midpoint which connects the point of intersection of the two outside flanges with the midpoint of the

curved flanges should be proportioned so that the minimum area of the diagonal stiffener is three-fourths of the flange area. Generally though, for ease in fabrication and as a matter of practicallity, the stiffener can be made the same thickness as the haunch flanges. The stiffeners at the points of tangency can be of nominal size.

4.2 Design Examples

The design procedure developed in this section can be illustrated by using two examples, one a connection for a rectangular portal frame and the other for a gabled frame. Consider first the design of the curved knee in Fig. 11.

In the assumed loading condition a plastic hinge forms at A, the point of tangency. Lateral support will be provided at the midpoint of the curved flange and at the points of tangency with the rolled section; therefore, the maximum allowable radius that is permitted is

$$R = 6b = 6(8) = 48 \text{ in.}$$

The plastic modulus required at point B is

$$Z = \frac{M}{\sigma_y} = \frac{1900}{33}\left(\frac{40+48 \sin 12°}{40}\right) = 72.0 \text{ in.}^3$$

The required thickness of the haunch flanges at B then becomes

$$t = \frac{13.0 - \sqrt{(13.0)^2 \left(\frac{8.0}{8.0-0.294} - \frac{4(71.2)}{8.0-0.294}\right.}}{2}$$

$$= 0.635$$

According to procedure 7

$$t \geq \frac{b^2}{2R}$$

$$t \geq \frac{8^2}{2(48)} = 0.67 > 0.635$$

Therefore increasing the thickness of the curved flange is necessary

$$\text{Use } \frac{11}{16} \text{ in. Plate } (0.69)$$

The diagonal stiffeners can be proportioned so the stiffener area is three-fourths of the flange area.

Hence,

$$t_s = \frac{3}{4}t = \frac{3}{4}(0.635) = 0.476 \text{ in.}$$

In view of the above required stiffener area, all radial stiffener requirements can be provided by $\frac{1}{2}$ in. plate.

The second design example will be a curved knee for a gabled frame. It will again be assumed that the prismatic sections have been selected so that the plastic hinge condition occurs at the point of tangency of the knee. The knee to be designed is shown in Fig. 12. From the loads and moments it is apparent that the critical section in the haunch occurs at 12 deg. from point A. Lateral support is provided at points A, C, and D. Hence, the maximum allowable radius becomes

$$R_{cr} = \frac{5b}{\alpha} = \frac{5(10.0)}{0.64} = 78 \text{ in.}$$

The radius used was

$$R = 72 \text{ in.} < 78 \text{ in.}$$

According to procedure 3 the required flange thickness of the haunch at point B is

$$t = \frac{d_z - \sqrt{d_z^2 \left(\frac{b}{b-w}\right) - \frac{4Z_z}{b-w}}}{2}$$

Since

$$Z_z = \frac{M_B}{\sigma_y} = \frac{9930}{33} = 301 \text{ in}^3$$

and

$$d_z = 26.91 + 72\,(1-\cos 12°) = 28.48 \text{ in.},$$

the thickness of the flange is

$$t = \frac{28.48 - \sqrt{(28.48)^2 \left(\frac{10}{9.5}\right) - \frac{4(301)}{9.5}}}{2}$$

$$= 0.76 \text{ in.} \quad (\text{Use } \tfrac{13}{16} \text{ in. Plate})$$

Next, from procedure 7

$$\frac{b}{t} \leq \frac{2R}{b}$$

$$\frac{10}{0.76} \leq \frac{2(72)}{10}$$

$$13.2 \leq 14.4$$

Thus the thickness provided is sufficient.

According to procedure 8 the radial stiffeners at the knee centerline and the points of tangency are considered. The required stiffener thickness is three-fourths of the flange thickness for the diagonal stiffener. Hence,

$$t_s = \frac{3}{4}\,(0.76) = 0.57 \quad (\text{Use } \tfrac{9}{16} \text{ in. Plate})$$

The stiffeners at the points of tangency will be arbitrarily made of the same material.

5. SUMMARY AND CONCLUSIONS

A method of proportioning curved corner connections based on simple plastic theory has been presented. The method is much simplier than any known elastic solution. The solution presented also considers the buckling characteristics of the component parts of the connection as well as the bending strength. The following summarizes the results of this theoretical study of curved haunched connections:

1. It has been shown that haunched corner connections with curved inner flanges may be analyzed by the plastic bending theory. The steps for proportioning curved haunched connections are given in Section 4.1.

2. Previous work has shown that failure of a built-up haunch has been through lateral buckling of the compression flange, even though adequate bending strength is provided. Plastic bending theory is based on the precepts of a structure stabilized against buckling. Therefore, a method has been developed for assuring sufficient rotation capacity such that failure does not prevent the haunch from transmitting the required plastic hinge moment.

3. The influence of axial force and shear on haunched connections can be considered in the same manner as for rolled sections. Generally the same modification that is applied to the adjacent rolled sections can be used.

4. The critical section in the knee is located at

$\beta = 12$ deg.

5. The effect of cross bending can be neglected if

$$\frac{b^2}{2Rt} \leq 1.$$

6. A design procedure has been formulated and design examples presented.

It should also be mentioned that a series of tests was conducted on connections designed and proportioned by the procedures developed herein, and the results showed good agreement[2].

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7. NOMENCLATURE

A = Area of cross section

A_f = Area of one flange

A_s = Area of stiffener

A_z = Area of section within the haunch

a = Distance between inflection point and end of haunched connection

b = Width of flange

c = Distance from neutral axis to extreme fiber

d = Depth of rolled section

d_z = Depth of section within the haunch

E_{st} = Strain-hardening modulus

F_s = Force resisted by diagonal stiffener

I_x = Moment of inertia about the x-axis

M = Bending moment

M_p = Plastic moment

M_{pr} = Applied moment at junction of the connection and the rolled section

M_z = Applied moment at section within the haunch

P = Normal force

p = Transverse force acting on the flange

R = Radius of curved haunch

r_x = Radius of gyration

s = Arc length: length of compression flange

T = External work

t = Thickness of flange

t_c	=	Thickness of compression flange
t_t	=	Thickness of tension flange
t_s	=	Thickness of the stiffener
u	=	Strain energy of bending
w	=	Thickness of web
y	=	Vertical distance between a point on the cross section and the neutral axis
Z	=	Plastic modulus
Z_r	=	Plastic modulus of rolled section
Z_z	=	Plastic modulus of a section within the haunch
α	=	Central angle between points of tangency of the curved connection
β	=	Angle between end of curved connection and critical section within the haunch
θ	=	Angle of rise in gabled frame
σ_y	=	Yield stress
σ_z	=	Stress in z-direction

8. FIGURES

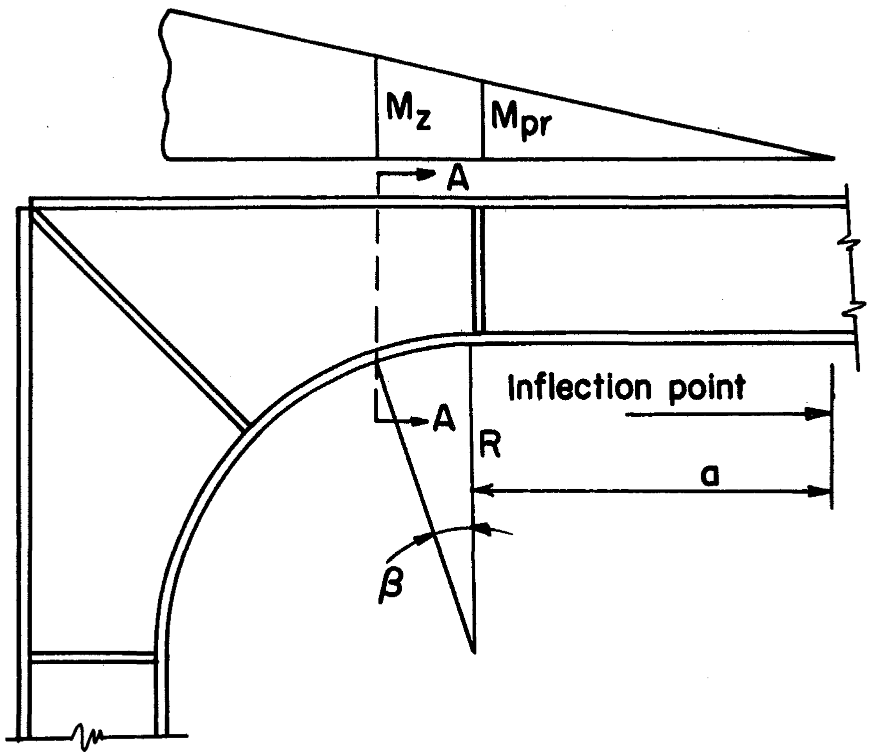

Fig. 1 General Proportions of Curved Knee and Assumed Moment Distribution

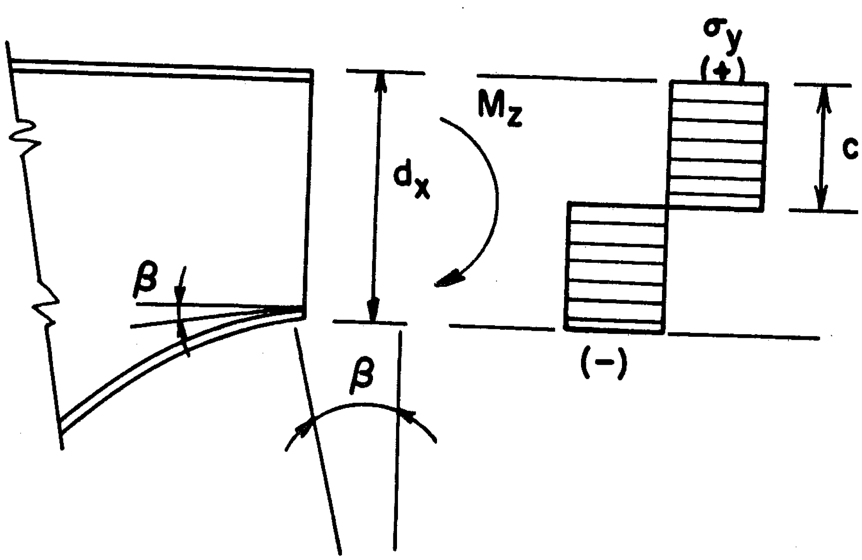

Fig. 2 Assumed Stress Distribution Within the Curved Knee

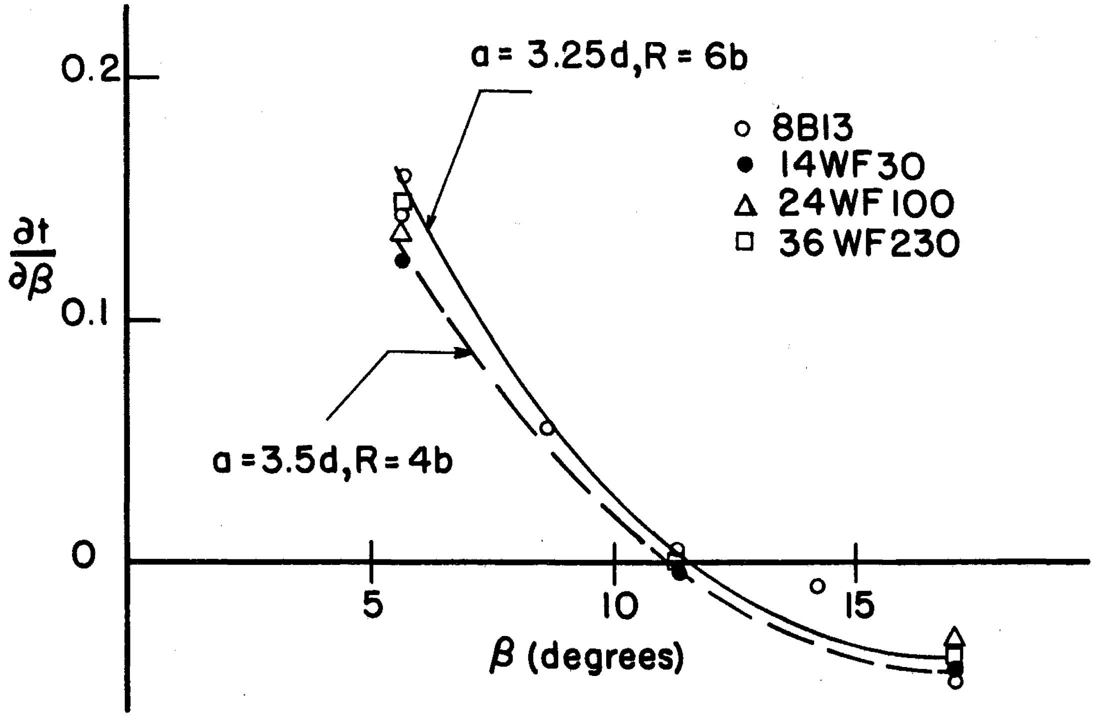

Fig. 3 Rate of Change of Flange Thickness With Respect to Angle β

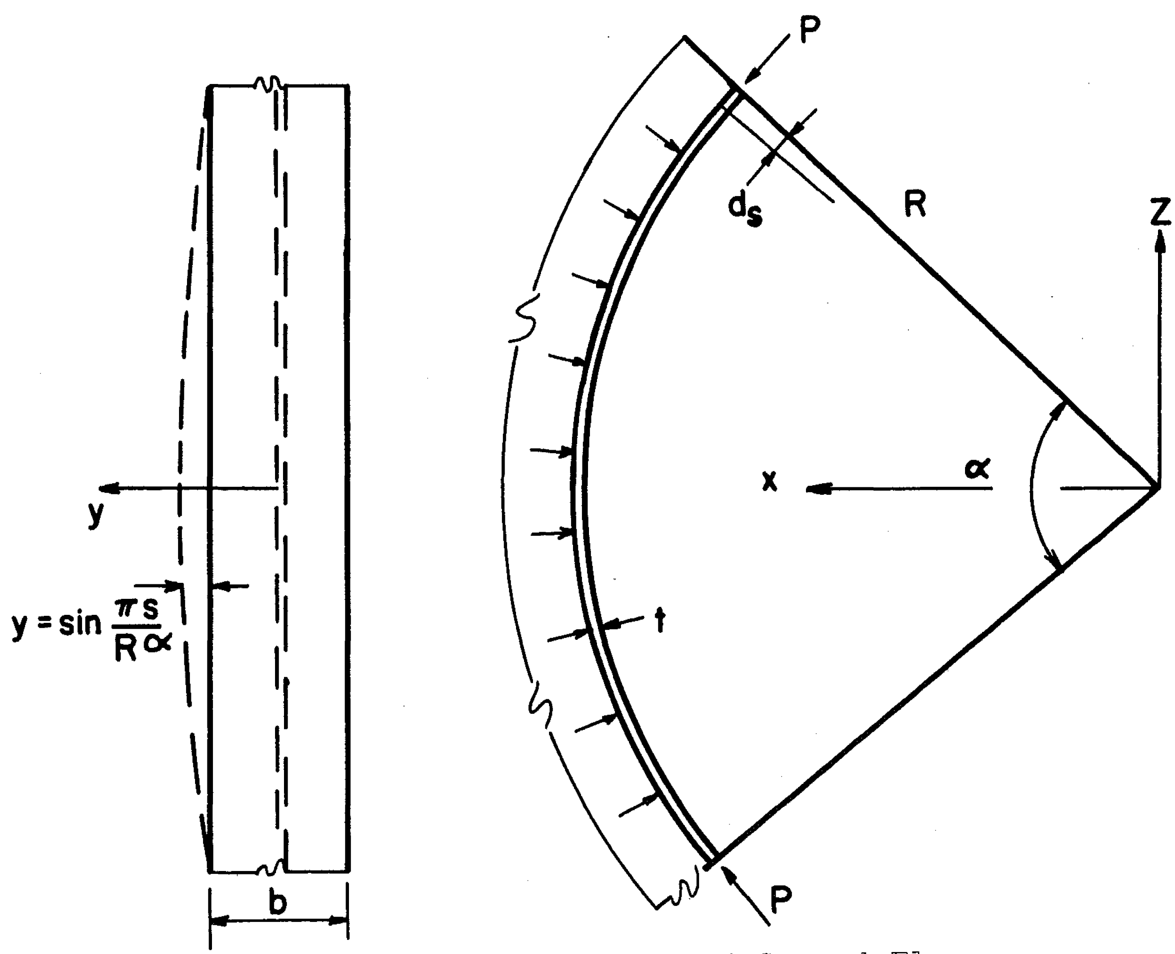

Fig. 4 Buckling Mode of Curved Flange

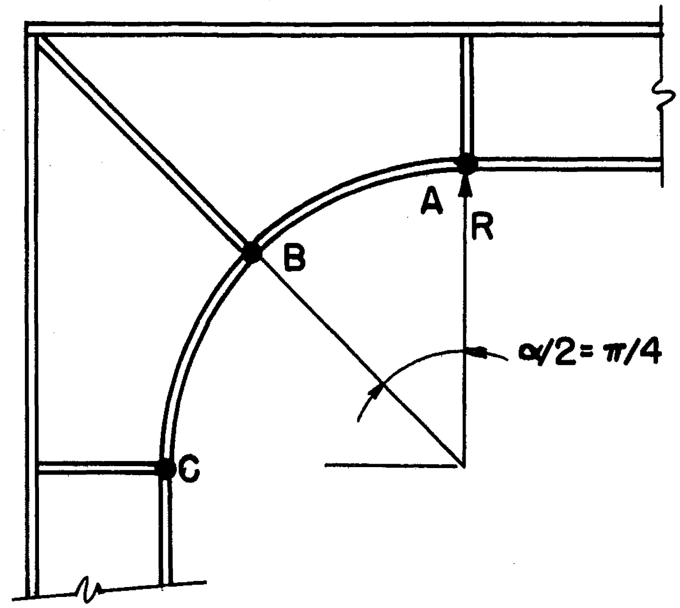

Fig. 5 Square-Corner Curved Knee Showing Points of Lateral Support

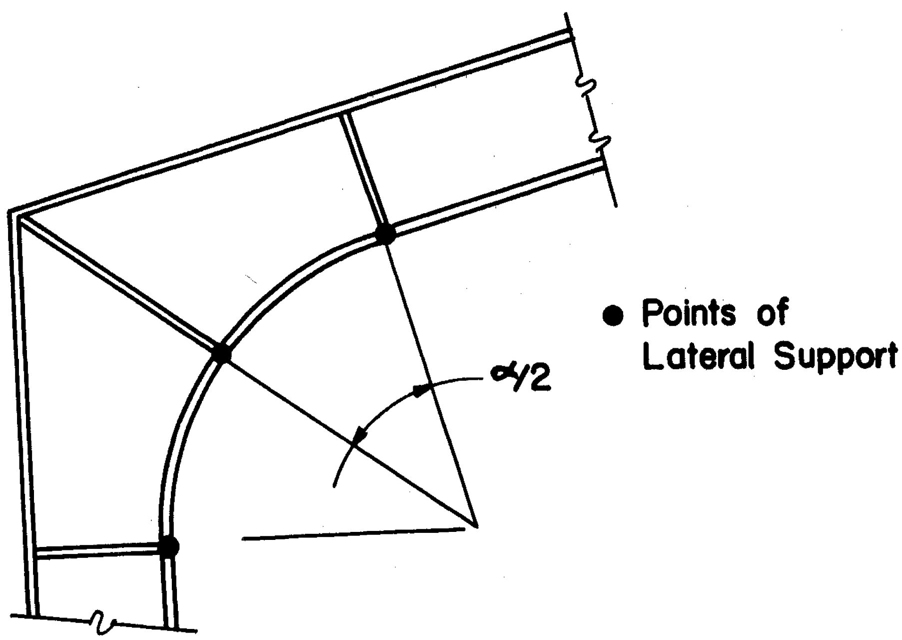

Fig. 6 Gable Frame Knee Showing Points of Lateral Support

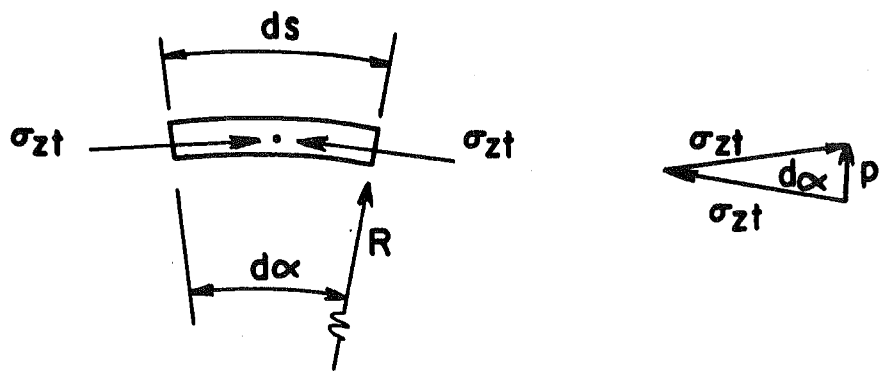

Fig. 7 Force in a Fiber of Radius "R"

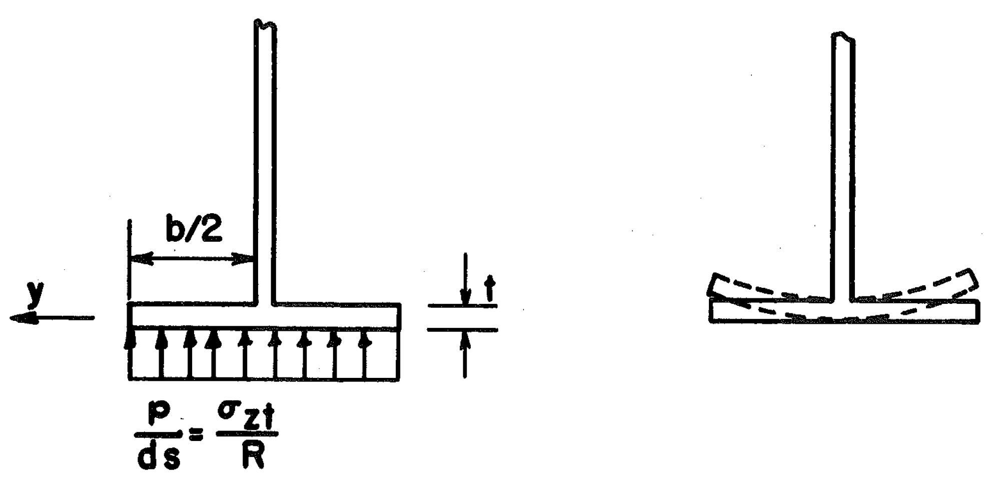

Fig. 8 Assumed Loading on Curved Flange and Resulting Deflection

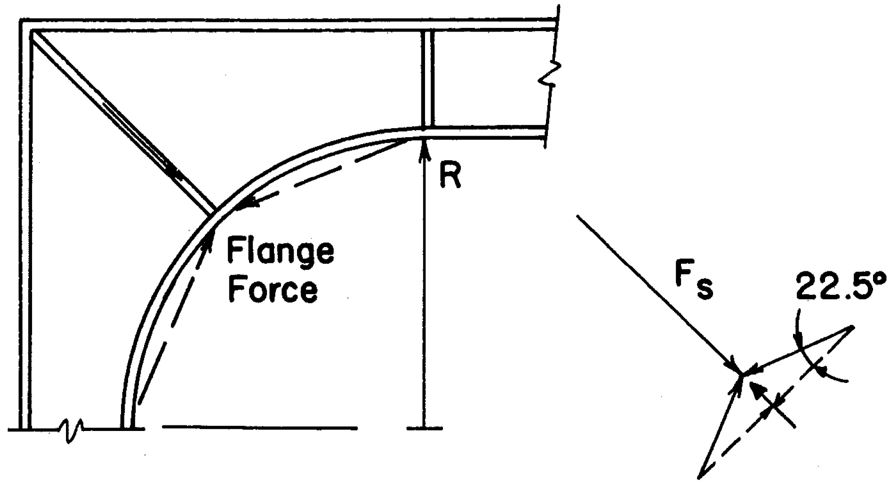

Fig. 9 Assumed Forces Acting on the Diagonal Stiffener in a Curved Knee

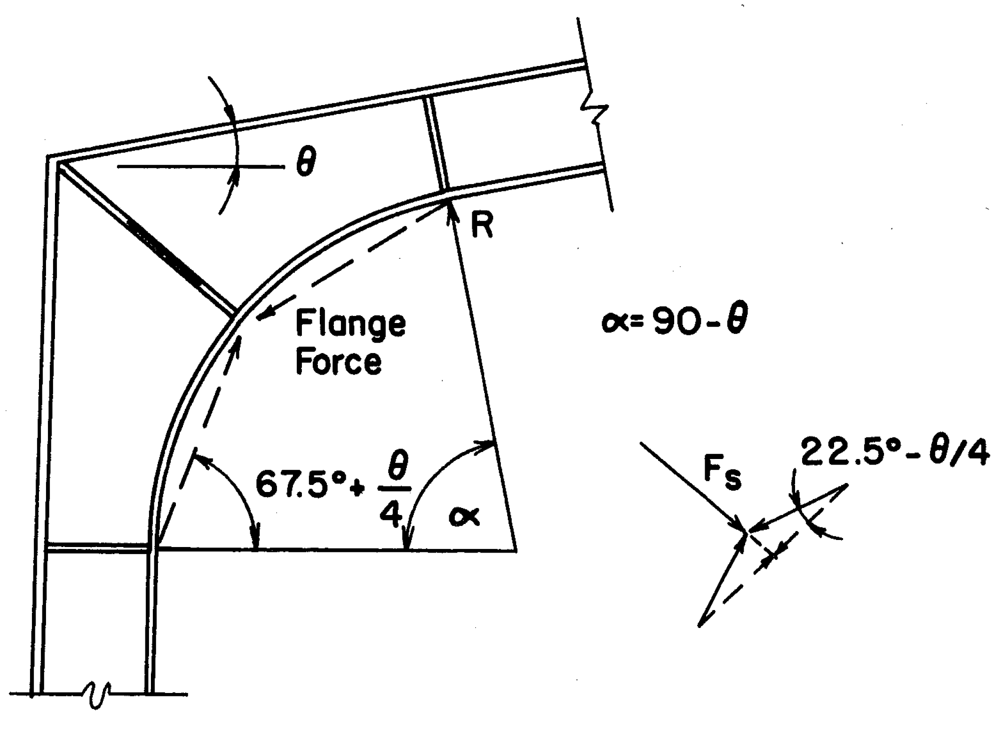

Fig. 10 Assumed Forces Acting on Diagonal Stiffener of Gable Frame Knee

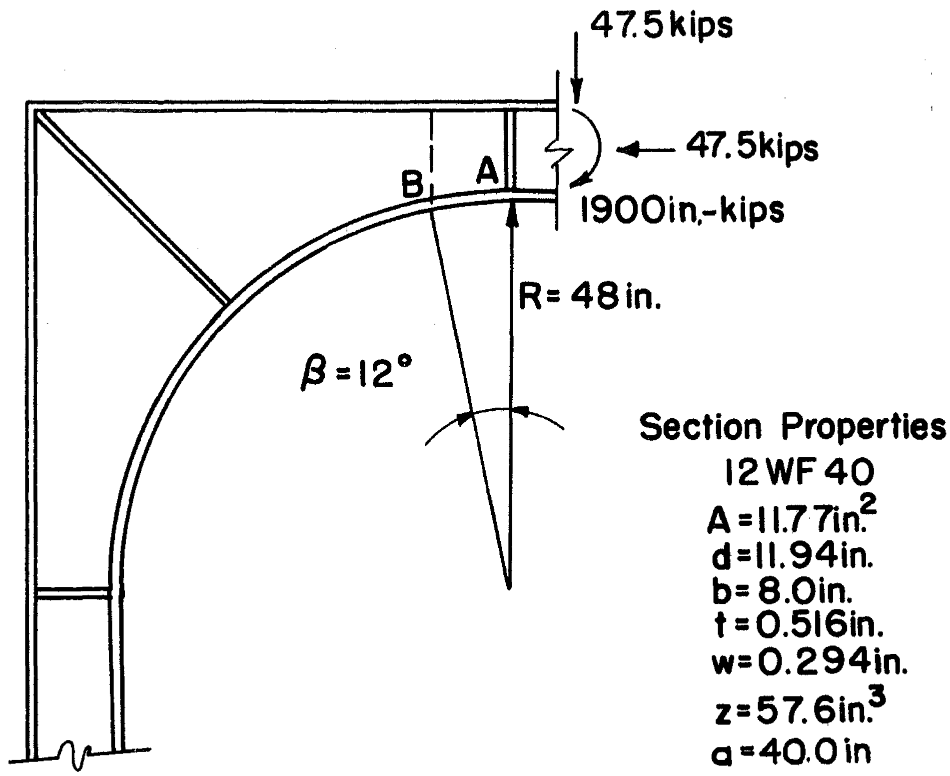

Fig. 11 Moment and Forces Acting on Knee

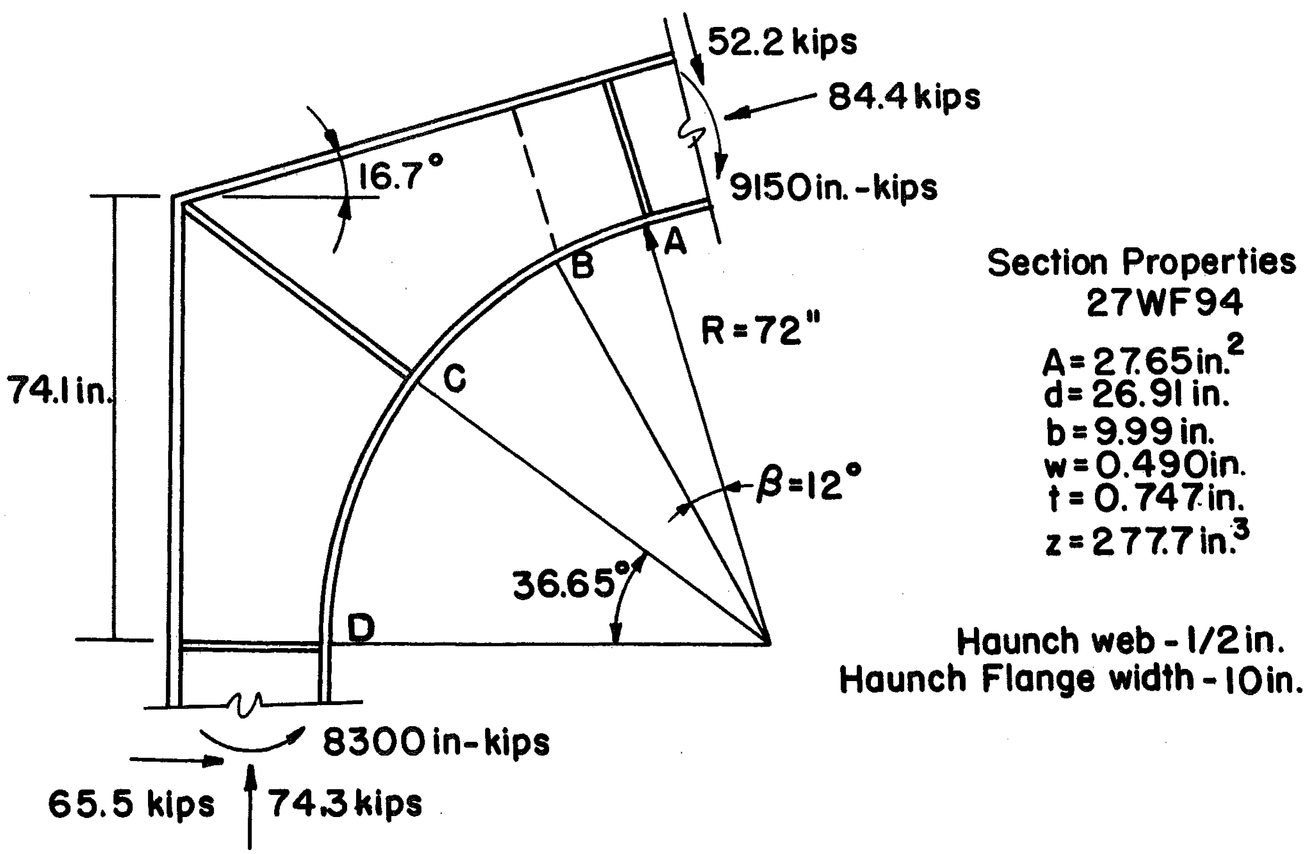

Fig. 12 Moments and Forces Acting on Gable Frame Knee

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